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WAVE DATA ANALYSES

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0.1 Abstract

The data collected over the course of the experiment must be analysed and converted into a form suitable for its intended use. Type of analyses range from simple to sophisticated. depending on the particular experiment and the needs of the researcher. In this study three main part of irregular wave data analyses are presented e.g. Time Domain (Statistical) Analyses, Frequency Domain (Spectral) Analyses and Wave Reflection Analyses. Random wave profile and definitions of representative waves, distributions of individual wave height and wave periods and spectra of sea waves are presented.

It is not completely a new work but is a producer which author has used for the analyses of the laboratory works during his Ph.D study under supervision of the co-authors. Because such a lecture is not presenting in the hydraulic engineering courses in the Iranian Universities it was decided to give such a lecture as a paper.

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0.2 Introduction

Waves are the most important phenomenon to be considered among the environmental conditions affecting maritime structures, because they exercise the greatest influence. Engineering application of the random wave concept is yet limited to a rather small number of researchers and engineers.

When the data acquisition and recording procedure is finished the data must be processed and presented. The statistical methods and error handling should be carried out, then the time series analysis can be done.

The first stage of data analysis following data verification and editing usually involves estimates of arithmetic means, variances, correlation coefficients,and other sample-derived statistical quantities. These quantities tell us how well our sensors are performing and help characterise the observed oceanographic variability. However, general statistical quantities provide little insight into the different type of signals that are blended together to make the recorded data. With the availability of modern high-speed computers, frequency-domain analysis has become much more centeral to our ability to decipher the cause and effect of oceanic change. The introduction of fast Fourier transform (FFT) techniques in 1960s further aided the application of frequency domain analysis methods in oceanography.

0.3 Time Domain (Statistical) Analyses

Time domain analyses provides representative statistical description of the wave data that are useful in coastal design, particularly in developing empirical relationships between important parameters used in design guidance. Time domain analyses includes nonlinear wave effects in a more direct manner than the linear techniques used in spectral analyses (Thompson and Long 1987). Consequently, this method of analyses is important for shallow waters waves. Time domain analyses is used primarily on records of sea surface elevation variations; however, the methods can be applied to any time-varying data records such as wave orbital velocities or pressures. Usually, the first step in analysing time series wave data is to calculate the mean water level and subtract it from the time series. Also, this is the time to remove any known linear trends in the data that were not been removed during data editing. The resulting "zero-meaned" time series is ready for analyses. Two types of time domain wave statistics can be obtained from time series data. The first type uses "n" evenly spaced values of sea surface elevation (η_i) to calculate statistical related to the sea surface elevations.

Typically calculated are the following statistics(Goda 19859):
mean Elevation (in cases when the mean has not already been removed)

$$\overline{\eta} = \frac{1}{N}\sum_{i=1}^{N}\eta_i \tag{0.1}$$

Variance (where η_{rms} the root-mean-squared sea surface elevation)

$$(\eta_{rms})^2 = \frac{1}{N}\sum_{i=1}^{N}(\eta_i - \overline{\eta})^2 \tag{0.2}$$

Skewness

$$\sqrt{\beta_1} = \frac{1}{(\eta_{rms})^3}.\frac{1}{N}\sum_{i=1}^{N}(\eta_i - \overline{\eta})^3 \tag{0.3}$$

Kurtosis

$$\beta_2 = \frac{1}{(\eta_{rms})^4}\frac{1}{N}\sum_{i=1}^{N}(\eta_i - \overline{\eta})^4 \tag{0.4}$$

Sea surface elevation can also be represented as a probability distribution, with the Gaussian (or normal distribution),i.e.,

$$p(\eta) = \frac{1}{\sqrt{2\pi}\eta_{rms}}exp[-\frac{1}{2}(\frac{\eta_i - \overline{\eta}}{\eta_{rms}})^2] \tag{0.5}$$

being the standard linear model for irregular waves. In the Gaussian distribution , skewness is zero and kurtosis is three, so these two parameters can give an indication of nonlinearity in the measured sea surface elevations.

The other type of time domain statistics that can be extracted from wave time serie records are related to individual waves contained within the record. Individual waves are determined by a "**zero-crossing**"analyses performed on a time series that has a zero mean. Each wave is defined as the sea surface elevation variation between two successive "down-crossings" or two successive "up-crossings" of the time series relative to the zero elevation. Wave height is taken as the vertical distance between the wave crest (highest elevation of the wave) and wave trough (lowest elevation of the wave).

A common problem with zero-crossing analyses occurs when the wave record contains high-frequency fluctuations that can be defined as individual waves during computer analyses unless some preventative action is taken. The usual solution is to filter out the high-frequency signal, leaving only those variations that represent gravity wave motion, and to establish a threshold elevation level that must be exceeded before another zero-crossing is counted. The filtering parameters and threshold criterion are subjective, and they may require adjustment to give results that compare well with hand-analyses wave records. One example can be nonlinear reflection analyses.

After individual waves have been delineated by zero-up crossing or zero-down crossing method, the wave heights and corresponding wave periods can be used to calculated various statistics and probability distributions. Commonly reported wave statistics determined from "N" waves include the following:

Mean Wave height

$$\overline{H} = \frac{1}{N}\sum_{i=1}^{N} H_i \tag{0.6}$$

Root-Mean Squared Wave Height

$$H_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} H_i^2} \tag{0.7}$$

Mean Wave Period

$$\overline{T} = \frac{1}{N}\sum_{i=1}^{N} T_i \tag{0.8}$$

Other popular statistical wave parameters include:

$H_{1/3}$ - significant wave height (which is the average of the highest 1/3 of the wave in the record)

H_{max} - maximum wave height in the record

$T_{1/3}$ - average period of the highest 1/3 of the wave in the record
The zero-crossing waves can also be represented as separate probability distributions of wave height and wave period or as a joint probability distribution of wave heights and periods. The Rayleigh probability distribution, given as

Rayleigh probability distribution

$$p(H) = \frac{2H}{H_{rms}^2} exp[-(\frac{H}{H_{rms}})^2] \tag{0.9}$$

has been shown to be a good first-order representation of irregular wave heights. The Rayleigh distribution has one "fitting" parameter, namely H_{rms}; whereas other distributions. such as the Weibull distribution, have more than one fitting parameter.

The distribution function of wave height, or the non-exceedence probability, is given as

$$P(H) = 1 - exp[-(\frac{H}{H_{rms}})^2] \tag{0.10}$$

and

$$Q(H) = -exp[-(\frac{H}{H_{rms}})^2] \tag{0.11}$$

$$lnQ(H) = -(\frac{H}{H_{rms}})^2 \tag{0.12}$$

where $Q(H) = P\{H_i > H\}$ (i.e. the probability that H_i exceeds a given wave height H). It appears from above equation that the Rayleigh-distribution can be sketched as a straight line in a $ln(Q) - (H/H_{rms})^2$ coordinate system which makes it easy to compare it with the distribution of wave heights in the time series. This distribution can be determined by using Weibulls formula which states that $Q(H_i)$ can be determined as

$$Q(H_i) = \frac{i}{number \ of \ waves + 1} \tag{0.13}$$

if the waves are stored with the largest first.

0.4 Frequency Domain (Spectral) Analyses

Wave data are analysed in the frequency domain using Fourier analyses techniques which assume the wave time series is a linear superposition of many sinusoidal components. Therefore, nonlinearities in the wave data are not easily identified as they are with time domain analyses.

Fourier analyses (or spectral analyses) of a wave record normally begins with some type of filtering or windowing to localise spillover of energy to other frequencies (Thompson and Long 1987). Then, the energy spectrum is estimated from the time series record using a Fourier transform technique. this procedure produces real and imaginary fourier components which are squared and summed to give a variance spectrum. Finally, that portion of the double-side spectrum in the positive frequency range is normalised so the area under the spectrum is equal to the total variance contained in the wave record.

The Fourier transformation can be applied to the entire time series record at once, or the record can be subdivided into several equal-length records and average is made of the spectra estimates from several adjacent frequencies to provide more stability to the spectral estimate. In the latter method, the spectral estimates at each frequency from all of the shorter segments are averaged. Both methods are commonly used for spectral wave analyses. If data are collected from several adjacent wave gages, then cross-correlation analyses can be performed on the data to provide indications of cross-correlation between the records.
aliasing can have a problem when the sampling rates are high enough (50-60 Hz) Goda (1985) recommend data sampling interval of 1/10 to 1/20 of significant wave period.
The zero-moment wave height is defined as

$$H_{mo} = 4\sqrt{m_o} \tag{0.14}$$

and

$$T_{o2} = \sqrt{\frac{m_o}{m_2}} \tag{0.15}$$

where m_o is the area under the energy spectrum

$$m_o = \int_{f_L}^{f_H} S_{\eta\eta}(f)df \tag{0.16}$$

and

$$m2 = \int_{f_L}^{f_H} f^2 S_{\eta\eta}(f)df \tag{0.17}$$

where $S_{\eta\eta}$ is the continuous energy spectral density, and f_L and f_H are lower and upper spectrum frequency bounds, respectively.

For narrow-banded spectra in deep water, H_{mo} is approximately equal to significant wave height ($H_{1/3}$); therefore, H_{mo} is approximately equal to "significant wave height". This can cause confusion because in shallow water, were nonlinearities cause a significant departure between H_{mo} and $H_{1/3}$. For this reason, care must be taken when using "significant wave height" to distinguish between the statistically based $H_{1/3}$ and the spectrally based H_{mo}.

Fourier analyses

The Fourier analyses is used to give a presentation of the frequency domain. This method is based on the fact that any periodic function can be written as a sum of harmonic components.

$$\eta(t) = a_o + 2\sum_{j=1}^{\infty}(a_j \cos(\frac{2\pi j}{T_M}t) + (b_j \sin(\frac{2\pi j}{T_M}t)) \tag{0.18}$$

where a_j and b_j are known as the Fourier coefficients that can be calculated as

$$a_j = \frac{1}{T_M} \int_0^{T_M} \eta(t) \cos(\frac{2\pi j}{T_M} t) dt \qquad j \geq 0 \tag{0.19}$$

$$b_j = \frac{1}{T_M} \int_0^{T_M} \eta(t) \sin(\frac{2\pi j}{T_M} t) dt \qquad j \geq 1 \tag{0.20}$$

(1.16) can also be written as sum of cosine-wave, i.e.

$$\eta(t) = \sum_{j=1}^{\infty} (A_j \cos(\frac{2\pi j}{T_M} t - \Phi_j) \tag{0.21}$$

where A_j is the amplitude and Φ_j is the phase of the j''th harmonic wave. A_j and Φ_j can be calculated as

$$A_j = 2\sqrt{a_j^2 + b_j^2} \tag{0.22}$$

$$\Phi_j = \arctan(\frac{b_j}{a_j}) \tag{0.23}$$

Often complex notation is used to combine the two equations in (1.19 and 1.20). This is done by defining the complex number X_j as

$$X_j = a_j - ib_j \tag{0.24}$$

and using

$$e^{-i(\frac{2\pi j}{T_M} t)} = \cos(\frac{2\pi j}{T_M} t) - i \sin(\frac{2\pi j}{T_M} t) \tag{0.25}$$

This leads to the following equation

$$X_j = \frac{1}{T_M} \int_0^{T_M} \eta(t) e^{-i(\frac{2\pi j}{T_M} t)} dt \tag{0.26}$$

In the case where elevation is known as a discrete time serie the period T_M is equal to $N.\Delta t$, where Δt is the time increment between the data in the time series and N is the number of data within a period T_M. (1.25) can then be rewritten to

$$X_j = \frac{1}{N} \sum_{k=0}^{N-1} \eta(t) e^{-i(\frac{2\pi jk}{N})} \tag{0.27}$$

which is known as the discrete Fourier transformation (DFT). the DFT returns N sets of Fourier coefficients, but only $N/2$ of these have physical meaning. This is due to the fact that the frequency f_k at $k = N/2$ is equal to the Nyquist frequency, i.e.

$$f_{N/2} = \frac{\frac{N}{2}}{N\Delta t} = \frac{1}{2\Delta t} = f_{Ny} \tag{0.28}$$

Since it is impossible to detect a harmonic component at a higher frequency than f_{Ny}, the Fourier coefficients found at $f \geq f_{Ny}$ cannot be regarded as are presentation of the harmonic components in the time series, but they are necessary if one wants to perform the inverse Fourier transformation (IDFT). The Fourier coefficients are symmetric (a) and asymmetric (b) around F_{Ny}

Once the Fourier coefficients are calculated, the amplitude of the individual harmonic components can be sketched as a function of their frequency f_j, as shown on Figure... It is often desired to sketch the spectral density instead of the amp-

Figure 0.1: The amplitudes as a function of the frequency. Δf is the frequency increment between two harmonic components.

litude. This can be done since the variance of a harmonic component is equal to $1/2A_2$. and the area below a spectrum must be equal to the variance. Therefore, the relationship between the amplitude and the spectral density can be expressed as

$$var_j = \frac{1}{2} A_j^2 \tag{0.29}$$

$$S(f_j) = \frac{var_j}{\Delta f} \tag{0.30}$$

The spectral density of $\eta(t)$ can then be sketched as shown on Figure.

The estimate of the spectral density is connected with uncertainty since the spectral density is estimated from the random process $\eta(t)$ which is usually not periodic. The time series is therefore usually divided into a number of blocks (n) each with a length of T_M, and the Fourier coefficients are then calculated as the average of the coefficients calculated in each block. The uncertainty of the Fourier analyses can be calculated as

$$V = \frac{\sigma}{\mu} = \frac{1}{\sqrt{M}} \tag{0.31}$$

where V is the variance coefficient, σ is the standard deviation, μ is the mean value and M is the number of block. Equation 1.31 is based on the assumption that $\eta(t)$ is a stationary gaussian random process. This assumption leads to the conclusion that the error of the estimate of the spectral density is χ^2-distributed and therefore the confidence interval for $S(f)$ can be written as

$$\left[\frac{\eta\hat{S}(f)}{\chi^2_{n;\alpha/2}} \leq S(f) \leq \frac{\eta\hat{S}(f)}{\chi^2_{n;1-\alpha/2}}\right] \tag{0.32}$$

where α is the fractile of the confidence interval (e.g. for a 90%) confidence interval $\alpha = 0.1$).

example...

0.5 Wave Reflection Analyses

Waves in coastal physical models are reflected by breakwaters, beaches, coastal structures, floating or submerged solid bodies, and model boundaries. The reflected waves interact with the incident waves and contribute to the characteristics of the wave field and the flow beneath the waves. Wave reflection is correctly reproduced in scaled laboratory studies with the additional complexity of reflected waves being re-reflected at the wave board unless some active absorption technique is employed.

In the following two analyses methods in frequency domain and time domain are presented for re-solving incident and reflected wave trains in two-dimensional laboratory wave flumes.

In the frequency domain methods it is assumed that the wave elevation to be a sum of regular waves travelling with different frequency and phase. The most common methods are given by Goda & Suzuki (1976) and Mansard & Funke (1980).

Method by Goda & Suzuki needs measurements of the incident and reflected waves in two distinct points. Hence by use of Fourier analyses the amplitude of the incident and reflected waves for a given frequency can by estimated.
The two point method has, however, certain limitations:

1-Limited Frequency Range
a) If the spacing between the probes is too greet, the coherency factor which estimates the relative phase stability in each spectral frequency band decreases as the frequency increase, thus making the calculation of reflections less reliable.

b) If the spacing is too short, then there is a loss of contrast in cross spectral analyses.

2-Critical Probe Spacing If the probe spacing "x" is such that x/L= n/2 (n=0,1,2,.., L= wave length), the values of reflections become indeterminate because the proposed equations have singularities at these values.

3-High sensitivity to errors in the measurement of waves duo to:
a) Transversal waves in the flume,
b) Non-linear wave interactions,
c) Harmonics duo to non-linearities,
d) Signal noise, measurement errors, etc.

As a natural extension to the method by Goda & Suzuki, Mansard & Funke presented a three-point method. Here an additional probe is taken into use which makes it possible to add an error to the measurements and hence minimise it in a least squares sense.

method by Mansard & Funke

The surface elevation in a two-dimensional wave field is assumed to be a summation of a number of waves, say N waves, i.e.

$$\eta(x,t) = \sum_{n=1}^{N} a_n cos(k_n x - \omega_n t + \Phi_n) \tag{0.33}$$

The wave elevation given by eq.(1.33) is separated into incident waves and reflected waves, but a noise function is added. This leads to:

$$\eta(x,t) = \sum_{n=1}^{N} a_{I,n} cos(k_n x - \omega_n t + \Phi_{I,n}) + \sum_{n=1}^{N} a_{R,n} cos(k_n x - \omega_n t + \Phi_{R,n}) \tag{0.34}$$

or

$$\eta(x,t) = \sum_{n=1}^{N} a_{I,n} cos(k_n x - \omega_n t + \Phi_n)$$
$$+ \sum_{n=1}^{N} a_{R,n} cos(k_n(x + 2x_R) + \omega_n t + \Phi_n + \theta_s) + \Omega(t) \tag{0.35}$$

where index p refer to probe number
θ is a phase shift from structure
$\Omega(t)$ is the noise function and express all kind of errors
x is the position of wave gages
x_R is the the position of wave gages to the structure
t is time
a_n is the amplitude
k_n is the wave number
ω_n is the angular frequency of the waves
Φ_n is the phase
η is the surface elevation relative to MWL

Inserting $x_{1,p}$ which is the distance from the 1st probe to p'th probe, eq. (1.35)can be written for one frequency as:

$$\eta_p = a_I cos(k(x_1 + x_{1,p}) - \omega t + \Phi)$$
$$a_R cos(k(x_1 + 2x_{R,1} - x_{1,p}) + \omega t + \Phi + \theta_s) + \Omega_p(t) \quad (0.36)$$

Fourier transformation of eq. (1.36) yields

$$\begin{aligned} A_p + iB_p &= a_I exp(ik(x_1 + x_{1,p}) + i\Phi) \\ &+ a_R exp(ik(x_1 + 2x_{R,1} - x_{1,p}) + i\Phi) + i(\Phi + \theta_s)) \\ &+ Y_p exp(i\rho_p) \end{aligned} \quad (0.37)$$

where A_p and B_p are the Fourier coefficients.
Let

$$Z_I = a_I exp(ik(x_1 + i\Phi) \quad (0.38)$$
$$Z_R = a_R exp(ik(x_1 + 2x_{R,1} + i(\Phi + \theta_s)) \quad (0.39)$$
$$Z_{N,p} = Y_p exp(i\rho_p) \quad (0.40)$$

where index N refer to noise.

$$A_p + iB_p = Z_I exp(ikx_{1,p}) + Z_R exp(-ikx_{1,p}) + Z_{N,p} \quad (0.41)$$

Minimising the noise function $\Omega_p(t)$ introduce in eq. (1.35) correspond to minimising the sum of squares of $\epsilon_p$for all p, i.e.

$$\sum_{p=1}^{3}(\epsilon_p)^2 = \sum_{p=1}^{3}(Z_I exp(ikx_{1,p}) + Z_R exp(-ikx_{1,p}) - (A_p + iB_p))^2 \quad (0.42)$$

should be minimised.
Assuming that the minimum of eq. (1.42) is achieved when both partial derivatives are zero, i.e.

$$\frac{\partial \sum_{p=1}^{3} \epsilon_p^2}{\partial Z_I} = \frac{\partial \sum_{p=1}^{3} \epsilon_p^2}{\partial Z_R} = 0 \qquad (0.43)$$

Instead of obtaining an exact solution a fitted solution is given by Mansard & and Funke, when the Fourier coefficients of the measured wave elevations are obtained by use of FFT analyses of the measurements.

In order to avoid singularities Mansard & Funke suggested
$x_{1,2} = \frac{L_n}{10}$
$\frac{L_n}{6} < x_{1,3} < \frac{L_n}{3}$
$x_{1,3} \neq \frac{L_n}{5} x_{1,3} \neq \frac{3L_n}{10}$

0.6 Conclusions

Time domain wave analysis were review, more direct manner of nonlinear wave effect was discussed. Useful statistical values were presented namely, mean value, variance, skewness, and kurtosis. Probability distribution of sea surface elevation and wave height were given. Problems with zero-crossing analyses were shown.

Spectral analyses and Fourier analyses were presented. Distribution of wave spectra density was given. Methods of reflection analyses were discussed and the limitation of the methods were shown. Suggestions for avoiding the singularities was given.

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